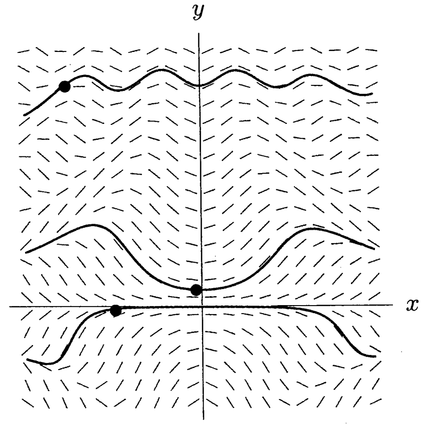

Figure 11.13: There's one and only one solution curve through each point in the plane for this slope field (Dots represent initial conditions)

In the language of differential equations, an initial value problem (that is, a differential equation and an initial condition) almost always has a unique solution. One way to see this is by looking at the slope field. Imagine starting at the point representing the initial condition. Through that point there is usually a line segment pointing in the direction of the solution curve. By following these line segments, we trace out the solution curve. See Figure 11.13. In general, at each point there is one line segment and therefore only one direction for the solution curve to go. The solution curve *exists* and is *unique* provided we are given an initial point. Notice that even though we can draw the solution curves, we may have no simple formula for them.

It can be shown that if the slope field is continuous as we move from point to point in the plane, we can be sure that a solution curve exists everywhere. Ensuring that each point has only one solution curve through it requires a slightly stronger condition.

Exercises and Problems for Section 11.2

Exercises

1. Sketch three solution curves for each of the slope fields in Figures 11.14 and 11.15.

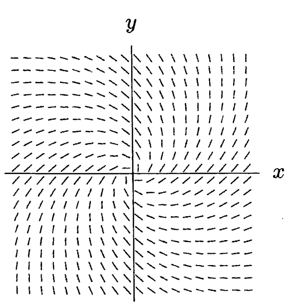

Figure 11.14

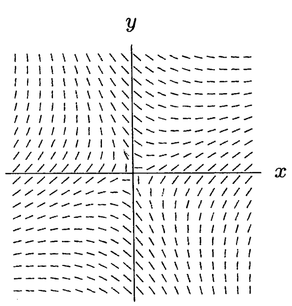

Figure 11.15

2. The slope field for the equation $y' = x(y-1)$ is shown in Figure 11.16.

(a) Sketch the solutions passing through the points (i) $(0,1)$ (ii) $(0,-1)$ (iii) $(0,0)$

(b) From your sketch, write down the equation of the solution with $y(0) = 1$.

(c) Check your solution to part (b) by substituting it into the differential equation.

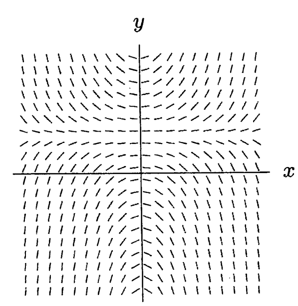

Figure 11.16

3. The slope field for the equation $y' = x + y$ is shown in Figure 11.17.

(a) Sketch the solutions that pass through the points
(i) $(0,0)$ (ii) $(-3,1)$ (iii) $(-1,0)$

(b) From your sketch, write the equation of the solution passing through $(-1,0)$.

(c) Check your solution to part (b) by substituting it into the differential equation.

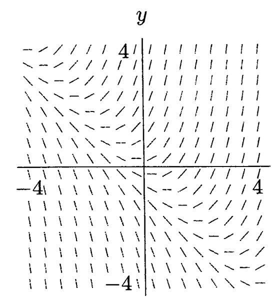

Figure 11.17: $y' = x + y$

Problems

4. One of the slope fields in Figure 11.18 has the equation $y' = (x + y)/(x - y)$. Which one?

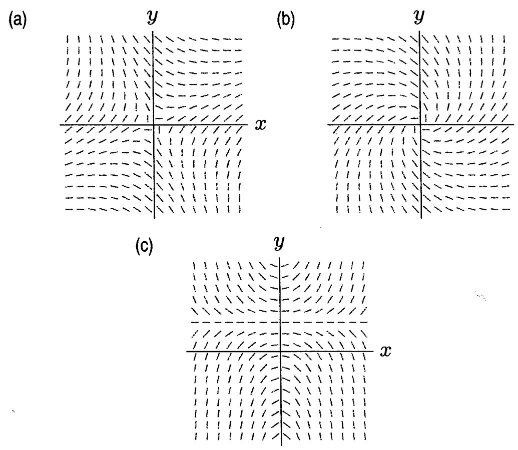

Figure 11.18

5. The slope field for the equation $dP/dt = 0.1P(10 - P)$, for $P \geq 0$, is in Figure 11.19.

(a) Plot the solutions through the following points:
(i) $(0,0)$ (ii) $(1,4)$ (iii) $(4,1)$
(iv) $(-5,1)$ (v) $(-2,12)$ (vi) $(-2,10)$

(b) For which positive values of P are the solutions increasing? Decreasing? What is the limiting value of P as t gets large?

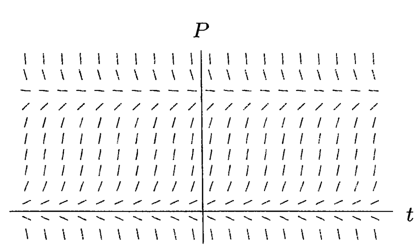

Figure 11.19

6. The slope field for $y' = 0.5(1 + y)(2 - y)$ is shown in Figure 11.20.

(a) Plot the following points on the slope field:
(i) the origin (ii) $(0,1)$ (iii) $(1,0)$
(iv) $(0,-1)$ (v) $(0,-5/2)$ (vi) $(0,5/2)$

(b) Plot solution curves through the points in part (a).

(c) For which regions are all solution curves increasing? For which regions are all solution curves decreasing? When can the solution curves have horizontal tangents? Explain why, using both the slope field and the differential equation.

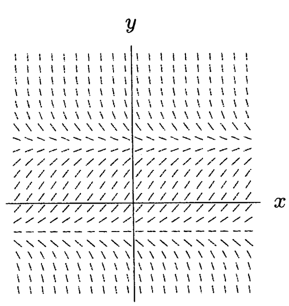

Figure 11.20: Note: x and y scales are equal

7. Figure 11.21 shows the slope field for the equation $y' = (\sin x)(\sin y)$.

(a) Sketch the solutions that pass through the points:
(i) $(0,-2)$ (ii) $(0,\pi)$.

(b) What is the equation of the solution that passes through $(0, n\pi)$, where n is any integer?